Heavy Lifting RC Model Aerodynamics

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Introduction

The Society of Automotive Engineers (SAE) sponsors an RC model airplane competition each year for college undergraduates called the SAE Design Challenge. Undergraduate teams are required to design, build and fly a model aircraft capable of lifting & flying with as much weight as possible under a set of design constraints. Outside help is not allowed except for general information purposes or for flight testing.

Videos of the competitions are available on You Tube and other places. While details vary from plane to plane, the overall designs are similar. They are almost all monoplanes with long high aspect ratio wings, highly cambered airfoils, tractor power, heavy-duty landing gear and of light construction.

This document examines the aerodynamic implications of the design criteria by means of an example. Since there are many possibilities for a design, one was chosen that illustrates the aerodynamic principles involved and the results are displayed in parametric fashion.

Design Constraints

The primary design constraints are as follows.

The sum of the maximum dimensions in the three axial directions must be less than or equal to 225 inches. (Height + Length + Width <= 225 inches).

The weight shall not exceed 62 pounds.

The aircraft must take off in less than 200 feet and land in less than 400 feet.

The airplane must fly at least one full circuit of the field after takeoff.

The engine must be an O.S. 61 Max glow engine. Prop is designer’s choice.

Fuel is 15% nitro glow fuel.

There are other requirements concerning building materials, weight bay access, etc. that affect the detail design but do not affect the aerodynamics very much.

Aerodynamic Design Factors

Level flight is not the limiting factor in meeting the goal of maximizing weight lifting capability. The real limitation is takeoff where the plane must climb against the pull of gravity. The heavier the plane, the more thrust & speed needed to get it off the ground. Equation one is the general equation for rate of climb of any aircraft.

Eq. 1 \[ R/C = (T - D) V / W \]

Where:

T is the thrust available (lbf)
D is the total drag (lbf)

V is the airspeed (ft/sec)

W is the weight (lbs)

The rate of climb decreases inversely with weight & dominates the results. There is an upper limit to thrust that decreases with airspeed due to the propeller characteristics. The drag is high at low speeds, decreases with increasing speed then increases with the square of the speed. Assuming a fixed weight, the rate of climb increases as speed increases, then flattens out and then decreases. The maximum climb rate decreases as weight increases.

The overall result is that the rate of climb is negative below some airspeed (it won't takeoff), then increases, flattens out, then decreases.

Minimizing takeoff distance is a little more complicated. The objective is to accelerate the plane from zero to some lift off velocity Vlo within the prescribed distance. Generally the velocity must be sufficient to reach maximum lift capability at rotation without stalling after rotation. Equation 2 provides an estimate of this velocity.

Eq. 2  \[ V_{lo} = \sqrt{\frac{2W}{S \cdot \rho \cdot 0.8 \cdot C_{l_{max}}}} \]

Where \( W \) is the weight, \( S \) is the wing area, \( \rho \) is air density & \( C_{l_{max}} \) is the maximum lift coefficient.

Increasing weight increases terminal takeoff speed while increasing wing area and \( C_{l_{max}} \) decreases take-off speed. This is the reason that the competition planes all have large wing areas and choose high lift airfoils.

The plane must accelerate from zero to \( V_{lo} \) within the prescribed distance. The acceleration varies during ground roll and is given by equation 3.

Eq. 3  \[ a = \frac{g}{W} \cdot [(T - D) - F_{c}(W - L)] \]

Where \( g \) is gravitational constant = 32.2 ft/sec^2

\( F_{c} \) is coefficient of rolling friction, typically 0.03 or so.

\( T \) is thrust at the given speed, \( D \) is the drag at that speed and \( L \) is the lift at that speed.

The takeoff distance is approximately \( TOD = \frac{V_{lo}^2}{2 \cdot \text{amean}} \)

Where \( \text{amean} \) is the acceleration at a speed of 0.7*\( V_{lo} \)

The acceleration decreases with increasing weight. Thrust is maximum at \( V = 0 \), then decreases somewhat to \( V_{lo} \). Drag is zero at \( V = 0 \), then increases with velocity, being maximum at \( V_{lo} \). Hence the effect of the \( T-D \) term is to be maximum at start and minimum at end. The last term \( W \) is constant but \( L \), lift, increases with velocity so that at the end \( L > W \).

So the general guidelines for design are to maximize lift (Area and lift coefficient \( C_{l_{max}} \)), maximize thrust, minimize drag and minimize weight not devoted to payload. There are interdependencies among these parameters that make the design process somewhat complex.
**Design Example**

**Dimensions**

To start choose the overall dimensions of the plane. Choose the wing span as 10 feet or 120 inches, leaving a combined $255 - 120 = 130$ inches for the other two dimensions. Choose the mean aerodynamic chord (MAC) as 12 inches, yielding a total wing area of 1440 square inches or 10 square feet. The aspect ratio is then 10:1. Takeoff speeds at maximum load will be around 35 mph, yielding a Reynolds Number of about 300,000, insuring that the profile drag is well behaved and near minimum.

**Air Foil**

The next step is to choose a high lift airfoil; a Selig 1223 highly cambered airfoil with a maximum Cl of about 2.2. Figure 1 is a drawing of this airfoil shape.

![Airfoil name: S1223](image)

**Figure 1. Selected airfoil shape**

Figure 2 is a graph of the airfoil Cl and Cd as a function of angle of attack. AoA = 0 is referenced to the dashed chord line in Figure 1. Note that at AoA = 0, the lift coefficient is about 1.1 and the drag coefficient is about .02.
Figure 2 S1223 Polars

Figure 3 is a graph of the ratio of lift coefficient to drag coefficient. It is relatively flat around the maximum value over a wide AoA range, indicating that the ratio of lift to drag is maximum over a wide operating range – a good thing.

Figure 3 Lift to Drag Ratio

Engine & Thrust

The engine must be an O.S.61 Max glow unit. It has a peak brake horsepower (BHP) of 2.0 at 15,000 rpm. The manufacturer’s recommended propeller is a 12 x 6. For the purposes of this example an APC Sport 12 x 6 is selected because wind tunnel data is available for similar props although at lower rpm than estimated here. When coupled
to this engine the propeller will operate at approximately 13,000 rpm when fully loaded. Figure 4 plots the resulting estimated thrust generated as a function of airspeed using the wind tunnel data. The equation shown on the graph matches the calculated values.

**Overall Drag**

The overall drag is composed of three major pieces. These parts are wing profile drag, wing induced drag and parasitic drag. The profile drag coefficient, $C_{do}$, is taken from Figure 2 for a given coefficient of lift, $Cl$. The induced drag coefficient is given by Equation 4.

$$C_{di} = \frac{0.318 \times Cl^2 \times (1 + \delta) }{AR}$$

Where $AR$ is the aspect ratio, in this case $AR = \text{span}^2 / \text{area} = 10^2 / 10 = 10$.

$\delta$ is a wing plan-form adjustment for non-elliptical shaped wings. For rectangular wing $\delta = 0.05$.

Parasitic drag is caused by the fuselage, landing gear, engine, tail and any other protruding object in the air stream. For purposes of this example the parasitic drag coefficient, $C_p = 0.01$.

The total drag is given by Equation 5.

$$\text{Drag} = (C_{do} + C_p + Cl) \times \rho \times V^2 \times S / 35 \text{  19  ounces}$$

$$P = 1.0 \text{ (air density at sea level and standard temperature)}$$

$V$ is the airspeed in mph

$S$ is the wing area in square inches
Rate of Climb and Angle of Climb Results

The results of the example analysis are shown in Figures 5 and 6. Also shown is the stall velocity as a function of airspeed Figure 7.

![Figure 5 Rate of Climb](chart1.png)

![Figure 6 Angle of Climb](chart2.png)
Start with Figure 7, stall speed. As the weight increases from 10 pounds to 50 pounds the stall speed increases from about 15 mph to about 30 mph. The forward airspeed must be greater than the stall speed to take off and maintain flight.

Figure 6, Angle of Climb, illustrates the results. The airspeed must be above the stall speed for any climb. The graph shows this as the abrupt jump from zero climb-angle at low speed to essentially the highest climb angle when the airspeed exceeds the stall speed. At this transition the wing is at its maximum angle of attack and greatest lift condition.

Another result is that the rate of climb is strongly influenced by the total weight. The plane will climb steeply at 10 pounds weight while will barely climb at 50 pounds. The best rate of climb at 10 pounds is about 35 mph while it is over 40 mph at 50 lbs.

Once the highest rate of climb or climb angle is reached, the rate or angle decreases with increased airspeed. Hence there is an optimum range of airscrews to maximize climb rate or climb angle.

**Takeoff Distance**

Takeoff distance is a function of weight, thrust, drag, lift, and wind speed. For this example it is assumed that the wing angle of attack is zero during the ground roll. The lift coefficient is about 1.2 & the drag coefficients are near the minimum (see Figure 2). The results are displayed in Figure 8.
The takeoff distance increases exponentially with weight increase. With no head wind this model reaches the 200 foot limit at about 45 pounds. A head wind of 5 mph increases the weight limit to 50 lbs. while a 10 mph headwind is under the limit at 50 lbs.

**Level Flight**

The engine thrust is sufficient to overcome the drag at airspeeds required to make lift equal drag for weights of at least 50 lbs. Figure 9 illustrates the excess thrust available as a function of air speed for a weight of 50 lbs. Note that the minimum speed is that of stall, in this case about 31 mph.
References

[www.students.sae.org/competitions/aerodesign/rules](http://www.students.sae.org/competitions/aerodesign/rules)


### Excel Spread Sheet VBA Code

Function \( t_{\text{max}}(v) \)

\[
t_{\text{max}} = -0.0007 \cdot v^2 - 0.0388 \cdot v + 11.203 \ \text{lbf}
\]

End Function

Function \( c_{\text{do}}(C_L) \)

\[
c_{\text{do}} = -0.0014 \cdot C_L^3 + 0.0253 \cdot C_L^2 - 0.0546 \cdot C_L + 0.0502
\]

End Function

Function \( \text{drag}(\text{weight, area, span, v}) \)

\[
\text{drag} = (c_p + c_{\text{do}}(C_L) + c_i) \cdot \text{sigma} \cdot v^2 \cdot \text{area} / 3519 / 16 \ \text{lbf}
\]

End Function

Function \( \text{climbrate}(\text{weight, area, span, v}) \)

\[
\text{climbrate} = v \cdot (t_{\text{max}}(v) - \text{drag}(\text{weight, area, span, v})) / \text{weight} \cdot 5280 / 3600
\]

End Function

Function \( \text{lift}(\text{weight, area, v, C_L}) \)

\[
\text{lift} = C_L \cdot v^2 \cdot \text{area} / 3519 / 16
\]

End Function
Function takeoff(weight, area, span, clmax, cto, vwind)
' computes takeoff distance in feet
' weight in lbs, area in square inches, span in inches, vwind in mph
' clmax is maximum lift coefficient, cto is lift coefficient during ground roll
' g = 32.2 'gravitational constant, ft/sec^2
Fc = 0.03 'coefficient of friction

vto = Sqr(weight * 16 * 3519 / (0.8 * clmax * area)) * 5280 / 3600 ' lift off speed, ft/sec
vmean = 0.7 * vto * 3600 / 5280 ' mean airspeed, mph

tmean = tmax(vmean + vwind) ' mean thrust, lbf

dmean = dragto(weight, area, span, vmean, cto, vwind) ' mean drag, lbf
lmean = lift(weight, area, vmean + vwind, cto) ' mean lift, lbs
amean = g / weight * ((tmean - dmean) - Fc * (weight - lmean)) ' mean acceleration, ft/sec^2

takeoff = vto ^ 2 / 2 / amean ' takeoff distance, feet

End Function

Function dragto(weight, area, span, v, cto, vwind)
' ground roll drag, weight in lbs, area in square inches, span in inches, v and vwind in mph

AR = span ^ 2 / area ' aspect ratio

sigma = 1.001 ' relative air density at sea level

w = weight * 16 ' converts to ounces

Cl = cto

ci = 0.318 * Cl ^ 2 * 1.05 / AR

cp = 0.01

dragto = (cp + cdo(cto) + ci) * sigma * (v + vwind) ^ 2 * area / 3519 / 16 ' lbf

End Function